

Final Technical Report

AD-A246 581



Office of Naval Research

Contract N00014-86-K-0140
1 January 1987 - 30 September 1990

STRUCTURE AND DYNAMICS OF CORRELATED
CLUSTER GROWTH PROCESSES

DTIC
ELECTE
FEB 28 1992
S D

Fereydoon Family
Principal Investigator

Department of Physics, Emory University
Atlanta, GA 30322

This document has been approved
for public release and sale; its
distribution is unlimited.

92-04756



92 2 25 080

STRUCTURE AND DYNAMICS OF CORRELATED CLUSTER GROWTH PROCESSES

Fereydoon Family

Department of Physics, Emory University, Atlanta, GA 30322

✓
The main thrust of our research has been the investigation of various aspects of the structure and dynamics of cluster growth processes. We have also used the concepts of cluster growth to investigate a number of other phenomena, including studies of the morphology of rough surfaces and the phenomenon of pattern formation. The following is a brief outline of the progress made in some of these studies.

DYNAMIC SCALING AND PHASE TRANSITIONS IN SURFACE GROWTH

➤ Self-affine fractal interfaces are generated in a variety of far-from-equilibrium processes, including fluid flow in porous media, vapor deposition, and corrosion, as well as in biological systems. Recently, there has been considerable interest in the study of stochastically growing interfaces (1), in the context of ballistic deposition, the Eden model, and Burger's equation. Much of this interest stems from the fact that in addition to their connection to processes of fundamental practical importance, such as thin film growth and interface dynamics in random media, these models exhibit non-trivial scaling behavior. The dynamics of the interface in these models has also been shown to be intimately related to a variety of other problems, including directed polymers in random media, the large-time behavior of randomly stirred fluids, and the evolution of Sivashinski flame fronts.

➤ The main quantities of interest in the scaling approach to interface dynamics are the exponents α and β which characterize the scaling of the interface width $W_L(t)$ with system size L and time t , respectively. Family and Vicsek [2] have proposed that the scaling of the interface width is expected to be of the form [2],

$$W_L(t) \sim L^\alpha f(t/L^{\alpha/\beta})$$

where the scaling function $f(x) \sim x^\beta$ for $x \ll 1$ and $f(x) \rightarrow \text{constant}$ for $x \gg 1$. For spatial dimension $d = 2$ (substrate dimension $d-1$), the results for ballistic deposition, the Eden model,

Dist	Special
A-1	

and Burger's equation agree giving $\alpha = 1/2$ and $\beta = 1/3$. For $d > 2$, however, there is still controversy over the values of the exponents as well as the universality of the various surface growth models. In addition, the dynamic renormalization group approach to Burger's equation has not been successful in predicting precise numerical exponents for $d > 2$. Therefore, the study of discrete models with continuously tunable parameters, which may be related to those in Burger's equation, and which can also be easily studied by numerical methods, can provide valuable insight into the dynamics of interface growth for $d > 2$.

We have carried out extensive numerical simulations of a restricted solid-on-solid surface growth model [3] in $d = 2 + 1$ dimensions which is a finite-temperature generalization of a model used by Kim and Kosterlitz (KK). Most of our simulations were carried out with a finite-density 'parallel' algorithm on the Naval Research Laboratory Connection Machine (NRL-CM2). With our algorithm, many sites are updated simultaneously and we have been able to generate surfaces using up to 10^{12} particles. This is by far the most extensive simulation of its kind and has enabled us to explore scaling regimes in system sizes that could not have possibly been studied on conventional computers.

We find that in our model there exists [3] a phase transition, as a function of a temperature-like parameter κ , in 2+1 dimensions. The surface width exponents cross over from the Kim-Kosterlitz values for small κ , to the low temperature values $\alpha \approx 0.25$, $\beta \approx 0.14$ for large κ . At the transition point κ_c , we find logarithmic growth and logarithmic dependence of the surface width on system size.

We have recently shown [4] that the origin of this unusual transition is intimately related with the connection between our model and the continuum Kardar-Parisi-Zhang (KPZ) equation of surface growth. In particular, the transition observed in our model is shown [4] to be due to a crossing of the corresponding nonlinearity parameter in the KPZ equation from negative to positive values. The critical point where logarithmic behavior is observed corresponds to the point at which the nonlinearity parameter vanishes. Our Monte Carlo simulation results for the growth velocity as a function of overall interface tilt support this conclusion.

NUMERICAL SOLUTION OF A CONTINUUM EQUATION FOR INTERFACE GROWTH IN 2+1 DIMENSIONS

Recently, there has been considerable interest in the study of rough surfaces and stochastically growing interfaces [1]. In particular, the continuum stochastic equation of Kardar, Parisi, and Zhang (KPZ) has been instrumental in providing a phenomenological basis for describing the dynamics of a variety of cluster growth models. For spatial dimension $d = 2$ (substrate dimension $d-1$), the results for ballistic deposition, the Eden model, and the KPZ equation agree giving $\alpha = 1/2$ and $\beta = 1/3$. However, for $d > 2$, there is controversy over the values of the exponents as well as the universality of the various surface growth models. In particular, the perturbative renormalization group approach to the KPZ equation has not been successful in predicting precise numerical exponents for $d > 2$. In addition, previous attempts to

solve the KPZ equation numerically in 2+1 dimensions were clearly flawed, because they yielded exponent values ($\alpha \approx 0.18$, $\beta \approx 0.09 - 0.15$) which were much smaller than those obtained for the discrete microscopic models ($\alpha \approx 0.33 - 0.40$, $\beta \approx 0.20 - 0.25$). Therefore, a reliable study of the continuum KPZ equation in 2+1 dimensions was needed to clarify the apparent disagreement between the results for microscopic models and the continuum equation.

We carried out [5] extensive large-scale numerical integrations of the Kardar-Parisi-Zhang (KPZ) equation for stochastic interface growth in 1+1 and 2+1 dimensions as a function of the non-linearity parameter ε . We found for the first time results for the growth exponents α and β close to those obtained for discrete models. In particular, we found that for large values of ε , the values of the exponents are close to the conjecture of Kim and Kosterlitz, indicating that the smaller values obtained previously are due to crossover effects.

Our study [5] has partially resolved the question of the existence of a phase transition in 2+1 dimensions, recently observed in discrete models. In contrast to the studies of the discrete models, our results do not show evidence of a phase transition in 2+1 dimensions for $\varepsilon \geq 1$. This indicates that either there is a fundamental difference between the discrete and the continuum models, or previous studies were not in the asymptotic regime. Clearly, further investigations of the discrete models is needed in order to fully settle this controversy.

GEOMETRICAL MULTIFRACTALITY OF DIFFUSION-LIMITED AGGREGATION CLUSTERS

→ Since the fractal scaling of the mass distribution in diffusion-limited aggregation (DLA) was discovered, it has been assumed that the structure of DLA clusters can be adequately described by a single exponent called the fractal dimension. Recent studies of measures defined on fractal substrates, however, indicate that many distributions in nature should be interpreted in terms of multifractal scaling, i.e., an infinite hierarchy of exponents is needed to account for the complicated structure of such measures. In particular, the growth probability distribution in diffusion-limited growth processes has been shown to exhibit multifractal scaling.

Much less is known about the multifractal properties of the *geometry* of the objects observed in fractal growth phenomena. Since the distribution of growth probabilities of diffusion-limited aggregates has been shown to be multifractal, it is natural to ask: Is DLA a geometrical multifractal or it can be described by a single fractal dimension?

We applied two different methods to address this question [6]. Using *large* off-lattice DLA clusters (consisting of one million particles) we obtained the left side of the mass multifractal spectrum $f(\alpha)$ directly from the number of particles within the boxes of the grid covering the clusters. Since the generalized dimensions corresponding to the negative moments $q < 0$ can not be determined reliably using box counting techniques we centered growing boxes around many randomly selected particles in the aggregates and determined the generalized dimensions from the scaling of the moments of the number of particles within boxes of various sizes. Our results support the picture that the distribution of mass in DLA clusters is multifractal, since the D_q values decrease monotonically with increasing q . Our calculations indicate [6] that

the commonly used method for the determination of the fractal dimension of DLA clusters yield $D_{q=2}$ which is somewhat *smaller* than the *true* fractal dimension $D_{q=0}$.

DROPLET GROWTH PROCESSES: SIMULATION, EXPERIMENT AND THEORY

The formation of a distribution of various size droplets is the characteristic feature of many systems from thin films and breath figures to rain, fog, clouds, foams and froths. In the past two year we have developed several models and analytical theories, and we have performed experiments on vapor deposited thin films in order to develop a better understanding of the kinetics of droplet growth and coalescence [7-13]. In general, droplet formation occurs either by spontaneous nucleation or by growth from a heterogeneously distributed nucleation centers, such as impurities. In homogeneous nucleation droplets can form and grow anywhere in the system. We have shown [7-11] that the droplet size distribution in homogeneous nucleation has a novel bimodal structure consisting of a monodispersed distribution of large droplets superimposed on a polydispersed distribution of smaller droplets. A scaling description for the evolution of the time dependent droplet size distribution and its moments is presented and it is found that the scaling predictions are in excellent agreement with the simulations. A rate equation approach similar to the Smoluchowski equation is also introduced for describing the kinetics of homogeneous droplet growth. The results of the simulations of the homogeneous nucleation model are also compared with the experiments on droplet growth in thin films obtained by vapor deposition of tin on sapphire substrates. It appears that this model captures the essential features of the distribution of droplets in the vapor deposition experiments.

We have also introduced a model for droplet growth by heterogeneous nucleation [8-10]. We show that in contrast to homogeneous nucleation, where the size distribution consists of a power-law distribution superimposed on a monodispersed distribution [7] at large sizes, the droplet size distribution has a monodispersed, bell-shaped form. We have shown that the origin of this difference lies in the fact that in homogeneous nucleation droplets are continuously fed into the system and this leads to a highly polydispersed stationary distribution. The exact exponents describing the size distribution and the growth law for droplet growth with heterogeneous nucleation are determined using scaling arguments. We have also developed a kinetic equation, similar to the Smoluchowski rate equation, for heterogeneous droplet growth. Using this approach we obtain a relation among the exponents and determine the scaling form of the reaction kernel.

SELF-ORGANIZED CRITICALITY AND AVALANCHE DYNAMICS IN A DROPLET GROWTH MODEL WITH SLIDING

What is the origin of the long-range spatial and temporal correlations that are commonly observed in many open dissipative systems? This question has attracted considerable attention in

recent years. The existence of a stationary state in a simple model of sand pile, has prompted Bak et al to propose that the type of scale -invariant avalanches that occur in the sand pile model might be related to the scaling and fractal behavior in other open systems. The existence of minimally stable states may explain the dynamics behind scaling and power law behavior observed in fractal systems. We have investigated [12,13] a model of droplet deposition and coalescence with avalanche.

In our model [12,13] we randomly add droplets to a system and once a given droplet reaches a critical mass, it falls along a preferred direction in the system and in the process coalesces and removes all the droplets that it comes in contact with. We have investigated the scaling properties of such systems using both lattice and off-lattice models. We find that the distribution of droplets and other properties of the system at the threshold obey scaling laws. However, unlike the sand pile model, there are no correlation between different avalanches. This suggests that the lack of a conserved parameter may be responsible for the difference between the behavior of our model and the sand pile model. We are currently continuing our investigations of the avalanche model using different dynamical rules.

FRACTAL STRUCTURE AND THE DEVELOPMENT OF HUMAN RETINAL VESSELS

The mechanism for the formation of retinal vessel patterns in the developing human eye is an unresolved question of considerable importance. The current hypothesis is based on the existence of a variable oxygen gradient across the developing photoreceptors which controls the release of angiogenic factors which in turn stimulate the growth of retinal vessels. This implies that the limiting step is a diffusion process. To test this hypothesis we have performed [14] a fractal analysis of human retinal vessels using the two-point correlation function and the mass-radius relation methods. We find that the human retinal vessels are self-similar with a fractal dimension of $D \sim 1.7$, which is expected for a diffusion limited process. This result supports the hypothesis that diffusion is the fundamental physical process underlying the formation of human retinal development.

ISING DYNAMICS WITH LONG-RANGE EXCHANGE

The Ising model has been used to simulate a wide variety of systems from magnets to fluids and binary alloys. Some of the most important questions about these systems involves their dynamical behavior. The classic picture of the dynamics of the Ising model developed by Glauber and by Kawasaki are based on spin-flip or spin-exchange between nearest neighbors. We have recently extended these approaches to include dynamics with long-range exchange [15]. In our model two spins are exchanged with a probability that depends on some power of the distance between them. Using a diagrammatic approach we have been able to determine exactly the dynamic exponents for the critical slowing down and for the domain growth. Long-range

dynamics [15] can be considered as a Levy flight generalization of the classical Ising dynamics which is based on spin diffusion. Our dynamical model [15] based on long-range exchange may be applied to experimental studies of phase separation in such systems as polymers solutions and polymer blends. It may also be relevant in the development of new fast algorithms for avoiding critical slowing down. There have already been some confirmations of our analytical results, and further numerical simulations are also underway.

INTERACTIVE PERCOLATION: A CORRELATED CLUSTER GROWTH MODEL

We have developed a model [16] to describe systems, such as atomic physisorption on surfaces, where interactions between neighbors can either enhance or deter absorption. Interactions are included in the standard model of site percolation by specifying two numbers, p_0 and p_1 , which are the probabilities of occupying a site on a lattice if none or at least one of the neighboring sites are occupied, respectively. Monte Carlo methods were used to simulate the irreversible growth of clusters starting from an empty two-dimensional square lattice, and continuing through the percolation threshold. The value of the percolation threshold p_c and other properties of the system depend on the parameter $r = p_0/p_1$. For $r=1$, the system corresponds to ordinary percolation. The limit $r \rightarrow 0$ gives the Eden model, and the limit $r \rightarrow \infty$ results in a checkerboard pattern. Several quantities were analyzed to determine the critical exponents D , ν , β , γ , and τ , including the threshold distribution width, the mean cluster size, the spanning cluster size, the cluster size distribution, and the cluster radius distribution. In all cases, the data are consistent with the exponents of ordinary percolation, indicating that all values of r lie in the same universality class.

RISING BUBBLE IN A HELE SHAW CELL

Fluid motion in a two dimensional Hele Shaw cell has received renewed interest in recent years. Attention has focused on the selection mechanism of a finger width that forms in the Hele Shaw cell. The problem of predicting the width of a steady state finger turns out to be mathematically similar to the selection mechanism of the dendrite growing in the undercooled melt; in the absence of surface tension, both problems possess a continuous family of solutions. Surface tension breaks this continuous family into a discrete set among which only one state is dynamically stable and thus selected.

A less studied but closely related problem is the motion of a finite bubble in a Hele Shaw cell. Consider a finite bubble in a Hele Shaw cell which is initially filled with fluid of higher viscosity. The question is: What will be the shape and the speed of the bubble, If we push the fluid from far left with the rate VW per second, where V is the velocity and W is the size of the wall? Saffman and Taylor worked out this problem in the absence of surface tension and discovered that for a given bubble area, the speed of the bubble is undetermined; its speed can

vary from V to infinity. Experimentally, however, unique velocity is selected. This problem is similar in spirit to the Saffman-Taylor problem, where the width of the finger is undetermined. We thus expect that a similar mechanism discovered in the Saffman-Taylor problem should work; surface tension breaks the continuous family into a discrete set.

We have recently studied [17] the problem of a rising bubble in a Hele-Shaw cell using the solvability theory. In the absence of surface tension, Saffman-Taylor solution contains two free parameters for a given bubble area. It was shown [17] that in the presence of surface tension a solution does not exist for $U > 2$ for any bubble area, where U is the dimensionless speed. We also made predictions for the shape and the speed of the symmetric bubble and obtained scaling relations between U and V as a function of the external parameters. Our results explain the recent experiment of Maxworthy on bubbles in a Hele-Shaw cell.

DYNAMICS OF RANDOM MEDIA: ANOMALOUS WAVE PROPAGATION AND DISPERSION NEAR THE PERCOLATION THRESHOLD

The dynamic properties of random media have received considerable attention in recent years. Much of this activity was prompted by the introduction of the concept of *fractons* by Alexander and Orbach, who made the observation that diffusion on fractals is anomalous. The importance of this anomalous behavior lies in the relation between diffusion and conduction that was first recognized by de Gennes. It was then possible to define two *independent* diffusion problems, namely the ant and the termite models of conduction. Because these two types of diffusion are decoupled from each other, we get two possible dynamics for the systems. This decoupling is clearly not a general result and as soon as the frequency is non-zero, the simple diffusion models are no longer valid.

As a first step in treating both propagation and dissipation in random systems, we have studied wave propagation close to the percolation threshold [18]. This might be an elastic wave for a sol-gel system, or an electromagnetic wave for a mixture of dielectric and metallic components. Because of the Kramers-Kronig relations, an extremely rich situation occurs in which the dynamics depends on both s and t , i.e. *the two anomalous transport laws are coupled*. At high frequencies propagation and dispersion are equally important and the frequency dependence of the dispersion relation is found to be anomalous and depends on *both* t and s . At low frequencies there is both propagation and dissipation above and below the threshold. Below the threshold, the group velocity is reduced and vanishes at the percolation threshold.

KINETICS OF REACTION-LIMITED CLUSTER-CLUSTER AGGREGATION

During the past few years a good understanding of both the structure and kinetics of diffusion-limited cluster-cluster aggregation (DLCCA) has been achieved. However, real systems only rarely come close to satisfying the conditions assumed in DLCCA (irreversible fast

aggregation to form rigid clusters). In most aggregation processes many encounters between pairs of clusters are required before two clusters are combined to form a larger cluster. In the limit where the number of encounters required for permanent bonding is very large (a condition frequently satisfied in practice), all possible bonding configurations (or at least a representative sample of them) can be explored before combination occurs. Under these conditions the aggregation process is limited not by cluster diffusion but by "chemical" details which determine how many collisions are required for cluster-cluster bonding. Although relatively few examples of chemically-limited cluster-cluster aggregation have been studied experimentally, it seems apparent that chemically limited aggregation is more common than diffusion limited aggregation. In dilute systems we might expect a crossover from chemically limited aggregation at short times (small clusters separated by relatively small distances) to diffusion limited aggregation at long times (large clusters separated by large distances) providing that other processes (such as mechanical instability and settling) do not intervene.

We have introduced and studied [19,20] several different models in order to investigate reaction-limited cluster-cluster aggregation and the crossover from diffusion-limited to reaction-limited aggregation. The results obtained from these models are consistent with each other, if finite size and finite concentration effects are taken into account. For reaction limited aggregation in three dimensions we find, in the case where the probability that two clusters will combine depends only on the time that they spend in contact with each other, that the mean cluster size $S(t)$ increases exponentially with time and that the cluster size distribution $N_S(t)$ decays as $N_S(t) \sim s^{-\tau}$ with τ having a value which is larger than 1.5. For the case where the probability that two clusters will combine depends only on the number of times they collide with each other, we find a power law growth in the mean cluster size, $S(t) \sim t^z$ with $z \sim 2.0-2.5$, and a cluster size distribution exponent τ close to 1.0. Our results [19,20] indicate that the approach to asymptotic behavior may be quite slow and that the effective fractal dimensionality of the clusters depends both on the aggregation kinetics and on the extent of aggregation. We find that if the rate of bonding between two clusters depends on their collision frequency then the exponent has a value close to 1 for the aggregation of small rigid clusters and close to 2 for the aggregation of large floppy clusters.

SCALING-GROUP FORMULATION OF MULTIFRACTALS

Scaling behavior is observed in many branches of physics, from percolation and aggregation to chaos and two-fluid flow in porous media. Fractals have provided a unified global description of scale invariance. It was first recognized by Mandelbrot in connection with turbulence that fractals can be partitioned into a set of scale-invariant subsets having an infinite number of scaling indices or fractal dimensions. Frisch and Parisi first coined the phrase "multifractals" for this kind of fractal set. Since Mandelbrot's first work on turbulence, there has been related work in turbulence as well as extensions and applications to aggregation, dynamical systems, percolation, localization, and polymer absorption. On the basis of the premise that

scaling is defined by a set of scaling transformations which form a simple product group, we developed [21] a scaling group formulation for simple fractals and multifractal sets. We showed that fractal dimension and other exponents are corollary to the scaling-group formalism as fractal dimension and other scaling exponents are generators of infinitesimal transformations.

PATTERN FORMATION IN DENDRITIC SOLIDIFICATION

The growth of dendritic crystals is a profound example among a wide range of spontaneous pattern forming phenomena in physics, chemistry, biology and engineering. The formation of snowflakes is perhaps the most fascinating and puzzling example of these processes. Although subject to intensive efforts, previous methods had not produced such regular and intricate dendritic structures as are found in nature. Therefore, the development of the deterministic growth model [22] which for the first time was capable of producing realistic dendritic patterns, including snowflakes, has been an important step in developing effective methods for understanding the formation of snowflakes and dendritic crystals.

The deterministic growth model [22] is based on direct solution of the diffusion equation on a regular lattice subject to proper boundary conditions far from the growing cluster and on the interface between the cluster and the outside. The temperature field is numerically determined by a relaxation technique such as the Gauss-Seidel over-relaxation method. For a fixed value of the surface tension, the temperature distribution at the surface are calculated from the Gibbs-Thomson condition, by numerically determining the surface curvature at each perimeter site. The interface is advanced by occupation of the perimeter sites, in analogy with the cluster growth model of Family, et al [23]. Since random growth processes cannot produce such symmetric patterns as snowflakes, our model is based on a deterministic growth rule: all perimeter sites having a normalized gradient larger than a fixed, but time dependent, value, are occupied.

The deterministic growth model [22] can produce practically all types of observed two-dimensional dendritic patterns by changing the surface tension and another adjustable parameter of the model. The various limiting cases include faceted growth, needle crystals and regular fractal structures. For intermediate values of the parameters, combinations of these patterns are obtained. By varying the parameters of the model, a great variety of qualitatively different patterns were found [22], many of which are qualitatively similar to real snowflakes.

SIMULATING VISCOUS FINGERING: A CLUSTER GROWTH APPROACH

The formation of patterns by moving interfaces is a common phenomenon in many fields of science and technology. Examples include the growth of snowflakes, directional solidification or the interface between two viscous fluid. A large class of these processes can be described by a diffusion-limited process. This means that the motion of the interface is governed by a spatial dependent field-like quantity (e.g., pressure) that satisfies the Laplace equation with moving

boundaries. In order to simulate such an interface and study pattern formation under these circumstances, we have used [24] an off-lattice version of the diffusion-limited aggregation model with surface tension. The approach of solving the discrete Laplace equation using random walkers is more effective than a direct solutions based on various finite difference methods. The patterns generated [24] in our simulations are very similar to those obtained in the experiments on fingering in the radial Hele Shaw cell experiments. We find a crossover from compact to fractal structure as the length scale is increased and the surface tension is decreased.

AGGREGATION OF ORIENTED ANISOTROPIC PARTICLES

The formation of clusters by the aggregation of small objects and by growth processes is a subject of considerable interest and of practical importance in physics, chemistry, biology, medicine, and engineering. We recently studied [25] the aggregation of oriented particles to form linear structures (rods). We had in mind the aggregation of particles and/or clusters with induced dipole moments in an external field, the aggregation of magnetic particles in an external magnetic field, and aggregation or polymerization processes in an ordered liquid crystal. Aggregation in magnetic fluids or ferrofluids is a particularly interesting example which is of considerable scientific and practical importance. We used Monte Carlo simulations and scaling theory to investigate clustering of anisotropic particles. The time-dependent cluster-size distribution in both two and three-dimensional systems was measured and was found to be well described by the dynamic scaling theory. The mean cluster size was found to diverge with time with the exponent z . The value of z was found to agree with the mean-field Smoluchowski result $z=1/(1-\gamma)$ in $d \geq d_c=2$ dimensions, where d_c is the upper critical dimension, and γ is the exponent describing the dependence of the diffusion coefficient on the cluster mass. Below d_c , we find that our results agree with the expression $z=d/(2-d)$. At high particle densities, a crossover from two or three dimensions to one-dimensional behavior was observed.

MANY-BODY EFFECTS IN TWO DIMENSIONAL OSTWALD RIPENING

The dynamics of a first-order phase transition is an example of the growth of order in a nonequilibrium system. A typical situation is initiated by a temperature quench such that the system is brought from a one-phase equilibrium state to a nonequilibrium state inside the coexistence curve. As a result, phase separation, such as the growth of droplets of the minority phase, occurs so as to minimize the surface energy. In this last stage, known as Ostwald ripening, the number of droplets is reduced and the process ends in equilibrium state of full phase separation. In many systems, including colloidal particles in solution, binary alloys, binary liquids and ionic gases, Ostwald ripening occurs by a condensation-evaporation mechanism.

Recent theoretical work predict that the droplet size distribution has a scaling form and the growth law of the critical droplet radius is of the form $R_c(t) \sim t^{1/3}$. However, numerical studies

indicate that the critical droplet radius grows as t^n with n less than $1/3$ at *early time*, but converging to $1/3$ in the long-time limit. The existence of a logarithmic correction has been suggested in two dimensions. The usual growth law $R_c(t) \sim t^{1/3}$ without a logarithmic correction, is based on the assumption of the existence of a screening length and a steady state of local concentration field of diffusing particles. It can be shown that there can be no local steady state without a screening length in two dimensions, because there is a logarithmic divergence of the Laplace equation in two dimensions.

We have recently carried out an analytic treatment which confirms the existence of a screening length in two-dimensional diffusion-controlled growth processes [26]. Our approach is based on a systematic perturbation theory, including the time dependence of local concentration field of diffusing particles. We find [26] that there is a screening length which is determined by a self-consistent condition. This results supports the analysis by Marqusee for treating two dimensional ripening, based on the assumption of the existence of a screening length.

DIFFUSION-ANNIHILATION IN ONE DIMENSION AND KINETICS OF THE ISING MODEL AT ZERO TEMPERATURE

The kinetics of diffusion-controlled annihilation in one dimension has been of interest for some time in the context of particle-antiparticle annihilation, binary reactions in one dimension, and exciton fusion kinetics in low-dimensional media. While the exponent characterizing the decay of the particle density in one dimension is well known and an exact solution has been given for an initial Poisson distribution in the continuum case, only recently have explicit solutions (for certain initial conditions) been given for diffusion-annihilation on a lattice. Because of the equivalence between domain walls in the Ising model and particles in diffusion-annihilation, it has been assumed that there exists an exact duality between the one-dimensional Ising model at zero temperature and diffusion-annihilation. In particular, Rácz has used this analogy to study the kinetics of diffusion-annihilation in the presence of sources. However, in the past, no direct comparison between the kinetics of the Ising model and diffusion-annihilation in one dimension had been made.

Recently, we derived [27] exact asymptotic expressions for the average domain size, wall density, and pair-correlation function for the one-dimensional kinetic Ising model at zero temperature, as a function of the initial magnetization (m_0). Our results turn out to be identical to known results for diffusion-annihilation in the case $m_0 = 0$. However, for general values of m_0 they differ. Monte Carlo simulation calculations for the domain wall distribution function as a function of m_0 were also carried out [27]. Again, agreement was found for the case $m_0 = 0$, while for $m_0 \neq 0$ our results depend on m_0 , in contrast to what is expected for the case of diffusion-annihilation. This demonstrates that the duality between the kinetic Ising model and diffusion-annihilation is only partial. Finally, we have studied the small x behavior of the domain size distribution scaling function $h(x)$ as a function of m_0 and have shown, for both the case of the Ising model and diffusion-annihilation, that the exponent τ is equal to 1.

REFERENCES

1. F. Family, "Fractal Concepts in Physics", Naval Research Reviews XL/XLI, 2 (1989); F. Family and T. Vicsek, "Simulating Fractal Aggregation", Computers in Physics 4, 44 (1990); F. Family, "Dynamic Scaling and Phase Transitions in Interface Growth", Physica A 164, *in press* (1990).
2. F. Family and T. Vicsek, "Scaling of the Active Zone in the Eden Process on Percolation Networks and the Ballistic Deposition Model", Journal of Physics A18, L75 (1985).
3. J. G. Amar and F. Family, "Phase Transition in a RSOS Surface Growth Model in 2+1 Dimensions", Physical Review Letters 64, 543 (1990).
4. D. A. Huse, J. G. Amar and F. Family, "Relationship Between a Generalized RSOS Growth Model and a Continuum Equation for Interface Growth", Physical Review A (Rapid Communications) 41, 7075 (1990).
5. J. G. Amar and F. Family, "Numerical Solution of a Continuum Equation for Interface Growth in 2+1 Dimensions", Physical Review A (Rapid Communications) 41, 3399 (1990).
6. T. Vicsek, F. Family and P. Meakin, "Multifractal Geometry of Diffusion-Limited Aggregates", Europhysics Letters 12, 217 (1990) and Family and P. Meakin, "Scaling of the Droplet Size Distribution in Vapor-deposited Thin Films: Experiment and Theory", Fractal Aspects of Materials: Disordered Systems, Materials Research Society, Extended Abstracts, p. 331 (1988).
7. F. Family and P. Meakin, "Scaling of the Droplet Size Distribution in Vapor-Deposited Thin Films", Physical Review Letters 61, 428 (1988).
8. P. Meakin and F. Family, "Scaling in the Kinetics of Droplet Growth and Coalescence: Heterogeneous Nucleation", Journal of Physics A22, L225 (1989).
9. F. Family, "Introduction to Droplet Growth Processes: Simulations, Experiments and Theory", in Proceedings of the NATO Advanced Study Institute on *Fluctuations and Patterns*, Edited by H. E. Stanley and N. Ostrowsky (Martinus Nijhoff, Dordrecht 1988).(25) F.
10. F. Family and P. Meakin, "Kinetics of Droplet Growth Processes: Simulations, Theory, and Experiments", Physical Review A 40, 3836 (1989).
11. P. Meakin and F. Family, "Droplet Growth Deposition and Coalescence: Computer Simulations", Fractal Aspects of Materials: Disordered Systems, Materials Research Society, Extended Abstracts, p. 335 (1988).
12. Z. Cheng, S. Redner, P. Meakin and F. Family, "Avalanche Dynamics in a Deposition Model with Sliding", Physical Review A.40, 5922 (1989).
13. F. Family, "Self-Organized Criticality and Avalanche Dynamics in a Droplet Growth Model with Sliding", Fractal Aspects of Materials - 1989, Materials Research Society, Extended Abstracts, p. 7 (1989).
14. F. Family, B. R. Masters, and D. E. Platt, "Fractal Pattern Formation in Human Retinal Vessels", Physica D 38, 98 (1989).

15. H. Hayakawa and F. Family, "Dynamics in a Long-Range Exchange Model", *Physica A* **163**, 454 (1990).
16. S. R. Anderson and F. Family, "Percolation in an Interactive Cluster-Growth Model", *Physical Review A* **38**, 4198 (1988).
17. D. C. Hong and F. Family, "Pattern Selection and Tip Perturbation for Finite Bubbles in the Hele Shaw Cell", *Physical Review A* **38**, 5253(1988).
18. M. Daoud, F. Family and D. C. Hong, "Dynamics of Random Media: Anomalous Wave Propagation and Damping near the Percolation Threshold ", *Journal of Physics A* **21**, L917 (1988).
19. P. Meakin and F. Family, "Structure and Dynamics of Reaction-limited Aggregation ", *Physical Review (Rapid Communications)* **A36**, 5498 (1987).
20. P. Meakin and F. Family, "Structure and Kinetics of Reaction-limited Aggregation ", *Physical Review A* **38**, 2110 (1988).
21. D. Platt and F. Family, "Scaling Group Formulation of Multifractals", *Physical Review Letters* **58**, 2786 (1987).
22. F. Family, D. Platt and T. Vicsek, "Deterministic Growth Model of Pattern Formation in Dendritic Solidification", *Journal of Physics A* **20**, L1177 (1987).
23. F. Family, T. Vicsek and R. Taggett, "Lattice-induced Anisotropy in a Diffusion-limited Growth Model", *Journal of Physics A* **19**, L727 (1986).
24. P. Meakin, F. Family and T. Vicsek, "Viscous Fingering Simulated by Off-lattice Aggregation", *Journal of Colloid and Interface Science* **117**, 394 (1987).
25. S. Miyazima, P. Meakin and F. Family, "Aggregation of Oriented Anisotropic Particles", *Physical Review A* **36**, 1421 (1987).
26. H. Hayakawa and F. Family, "Many-body Effects in Two-dimensional Ostwald Ripening", *Physica A* **163**, 491 (1990).
27. J. G. Amar and F. Family, "Diffusion-Annihilation in One Dimension and Kinetics of the Ising Model at Zero Temperature", *Physical Review A* **41**, 3258 (1990).